Student Centered Instructional Strategies

June 17, 2015
Learning Outcomes

Teachers will examine various instructional strategies related to student-centered mathematics.

● Task based learning
● Cognitively Guided Instruction
● High Level Questioning
● Number Talks
Revisit SMP 7 & 8

**SMP 7**

Look for and make use of structure.

Mathematically proficient students look closely to discern a pattern or structure.

**SMP 8**

Look for and express regularity in repeated reasoning.

Mathematically proficient students notice if calculations are repeated, and look for both general methods and shortcuts.
How many squares do you see?

The Squares task

How many total squares can you find in the drawing below?

SMP 7 or SMP 8?

SMP 7 or SMP 8?

Extending the Squares task

How many total squares can you find in the drawings below?
SMP 2: Reason *abstractly* and quantitatively

Cari is the lead architect for the city’s new aquarium. All the tanks in the aquarium will be rectangular prisms where the side lengths are whole numbers. . . . Cari knows that a certain species of fish needs at least 240 cubic feet of water in [its] tank. Create three separate tanks that hold exactly 240 cubic feet of water.

Source: http://www.illustrativemathematics.org/illustrations/1308
What is Task-Based Learning?
Task Based Learning

What do we believe about task based instruction and learning?

• What is it?
• Why do we need to use it?

Post your responses to
https://todaysmeet.com/SMath2015
Task Based Learning

We believe.....

- important
- pulls in math practices
- engaging
- authentic
- applicable
- force thinking
- develops problem-solving
- critical thinking
- encourages connections
- open-ended
- collaboration

- exploration & investigation
- multiple strategies
- application of knowledge
- real world connections
- strategic problem-solving
- purpose
- sense-making strategies
- productive, positive struggle
- multiple entry points
- used as formative assessment

We believe...
Task Based Instruction
Task Based Learning

Watch this video of a class working on a fraction task. Look for:

- What is the teacher’s role? What are her responsibilities?
- What is the student's' role? What are they responsible for?
- What are the “rules” or norms for how math is done in this classroom?
<table>
<thead>
<tr>
<th><strong>Teacher’s Role</strong></th>
<th><strong>Students’ Role</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>• engaged the students with an authentic setting/task</td>
<td>• defend thinking</td>
</tr>
<tr>
<td>• facilitated discussion</td>
<td>• participate in discussion (adding on, listening to others)</td>
</tr>
<tr>
<td>• used talk moves (clarification, revoicing, agree/disagree)</td>
<td>• take in information</td>
</tr>
<tr>
<td>• provided visual</td>
<td>• teacher - shared information</td>
</tr>
<tr>
<td>• addressed misconceptions (but not directly)</td>
<td>• passionate about beliefs</td>
</tr>
<tr>
<td>• listened and redirected with questions</td>
<td>• use multiple tools</td>
</tr>
<tr>
<td>• pushed to use symbols, equations</td>
<td>• question and challenge others’ thinking</td>
</tr>
<tr>
<td>• allowed students to conduct their own discussion, but intervened when necessary</td>
<td>• collaborate as a group</td>
</tr>
<tr>
<td>• instilled confidence in students previously</td>
<td>• create their own understanding</td>
</tr>
</tbody>
</table>

**Rules or Norms**

- Listen to others.
- Question each other and the teacher.
- Respectful of others’ ideas.
- Participate in discussion. No opt out of discussion. All students got to share
- Expectations were clear - working together, volume level, how to ask questions, participation
- Students knew the difference in questions/challenges versus bullying.

“Lots of work has gone into the classroom culture.”
Task Based Instruction

Based on the video, let’s refine our definition. What are the most important points to remember about TBI?

What trade vocabulary might we use when talking about TBI?
Borrowed ideas about TBI

Tasks provide opportunities:

- for kids to investigate, assess, and refine mathematical rules and generalizations
- to work with equivalence and place value, and to build number sense
- to contextualize mathematics and learn how context influences how we think about and deal with numbers
- to engage in math discourse and share ideas, develop a math identity
- to build reasoning and sense making skills by working with a variety of manipulatives and tools

Suggestions adapted from Beyond Pizzas and Pies by McNamara & Shaughnessy (2015)
Borrowed ideas about TBI

Tasks must:

● have a mathematical goal (CCSSM)
● allow students to make connections to content they already know
● allow for multiple entry points
● allow for multiple solution methods
● encourage students to reason about math and allow them to communicate mathematically

Build a checklist

What are the essential elements of a task?

- open-ended
- specific goals
- communication/discourse
- connections to prior knowledge
- collaboration
- differentiation (all students)
- multiple paths to the solution
- real-world scenarios
- cooperative engagement & investigation

- multiple strategies
- reasoning
- skill
- quality
- positive struggle
- encourages
- critical thinking
- authentic
- problem-solving
- purposeful
- leading role
- explain, analyze, question, defend thinking
- variety
- hands-on
- safe and structured
Impact on Instruction

- What impact do you think task based instruction can have on student learning?
How can we make instruction open-ended?

- AMC activities - Rearranging Cubes
  - How many different ways can you rearrange 36 cubes?
  - Place Value Activities - Looking for Patterns
  - Counting
Group Discussions

“1 ÷ \( \frac{2}{3} \) can be a task if you focus on sense making.”

(Jo Boaler, March 27, 2015 at NCCTM Annual Meeting, Greensboro, NC)
Task Based Instruction Resources

- Illustrative Mathematics
- NRICH
Planning Time

- Work within your group to focus on the standard from the previous day.
- What are some ways in which you could add a task or open-ended activity?
Cognitively Guided Instruction and a Framework for Learning
What is Cognitively Guided Instruction?

10.4: Ms. Dominguez's class engages with each other's idea
CGI Framework

How does this framework support and/or relate to Task Based Learning?
CGI Framework - Activity

CGI supports Tasked Based Learning because it requires teachers to:

● Anticipate where student thinking may go.
● Have questions to drive student thinking and understanding forward
● Have a plan for and an idea of student struggles with new concepts.
CGI Framework - Activity

In a group of two or three look at the problem(s) on your table. Solve the problem and then think about those three teacher moves that will support student thinking.

- Anticipate where student thinking may go.
- Have questions to drive student thinking and understanding forward.
- Have a plan for and an idea of student struggles with new concepts.
CGI Framework - Activity

Share what your group noticed about the problems and what you would do with students.
CGI Framework - Problems

- CGI is not just using the problem types.
- But we need to be aware of the different types and in what grade level should they be introduced.
### Glossary

#### Table 1: Common Addition and Subtraction Situations

<table>
<thead>
<tr>
<th></th>
<th>Result Unknown</th>
<th>Change Unknown</th>
<th>Start Unknown</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Add to</strong></td>
<td>Two bunnies sat on the grass. Three more bunnies hopped there. How many bunnies are on the grass now? (2 + 3 = ?)</td>
<td>Two bunnies were sitting on the grass. Some more bunnies hopped there. Then there were five bunnies. How many bunnies hopped over to the first two? (2 + ? = 5)</td>
<td>Some bunnies were sitting on the grass. Three more bunnies hopped there. Then there were five bunnies. How many bunnies were on the grass before? (? + 3 = 5)</td>
</tr>
<tr>
<td><strong>Take from</strong></td>
<td>Five apples were on the table. I ate two apples. How many apples are on the table now? (5 - 2 = ?)</td>
<td>Five apples were on the table. I ate some apples. Then there were three apples. How many apples did I eat? (5 - ? = 3)</td>
<td>Some apples were on the table. I ate two apples. Then there were three apples. How many apples were on the table before? (? - 2 = 3)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th><strong>Put Together/ Take Apart</strong></th>
<th>Total Unknown</th>
<th>Addend Unknown</th>
<th>Both Addends Unknown</th>
</tr>
</thead>
<tbody>
<tr>
<td>Three red apples and two green apples are on the table. How many apples are on the table? (3 + 2 = ?)</td>
<td>Five apples are on the table. Three are red and the rest are green. How many apples are green? (3 + ? = 5, 5 - 3 = ?)</td>
<td>Grandma has five flowers. How many can she put in her red vase and how many in her blue vase? (5 = 0 + 5, 5 = 5 + 0, 5 = 1 + 4, 5 = 4 + 1, 5 = 2 + 3, 5 = 3 + 2)</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th><strong>Compare</strong></th>
<th>Difference Unknown</th>
<th>Bigger Unknown</th>
<th>Smaller Unknown</th>
</tr>
</thead>
<tbody>
<tr>
<td>(&quot;How many more?&quot; version): Lucy has two apples. Julie has five apples. How many more apples does Julie have than Lucy? (2 + ? = 5, 5 - 2 = ?)</td>
<td>(Version with &quot;more&quot;): Julie has three more apples than Lucy. Lucy has two apples. How many apples does Julie have?</td>
<td>(Version with &quot;more&quot;): Julie has 3 more apples than Lucy. Julie has five apples. How many apples does Lucy have? (5 - 3 = ?, ? + 3 = 5)</td>
<td></td>
</tr>
<tr>
<td>(&quot;How many fewer?&quot; version): Lucy has two apples. Julie has five apples. How many fewer apples does Lucy have than Julie? (2 + ? = 5, 5 - 2 = ?)</td>
<td>(Version with &quot;fewer&quot;): Lucy has 3 fewer apples than Julie. Lucy has two apples. How many apples does Julie have? (2 + ? = ?, 3 + 2 = ?)</td>
<td>(Version with &quot;fewer&quot;): Lucy has three fewer apples than Julie. Julie has five apples. How many apples does Lucy have?</td>
<td></td>
</tr>
</tbody>
</table>

K: Problem types to be mastered by the end of the Kindergarten year.

1st: Problem types to be mastered by the end of the First Grade year, including problem types from the previous year. However, First Grade students should have experiences with all 12 problem types.

2nd: Problem types to be mastered by the end of the Second Grade year, including problem types from the previous years.
Table 2 Common multiplication and division situations

<table>
<thead>
<tr>
<th>Unknown Product</th>
<th>Group Size Unknown (&quot;How many in each group?&quot; Division)</th>
<th>Number of Groups Unknown (&quot;How many groups?&quot; Division)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$3 \times 6 = ?$</td>
<td>$3 \times ? = 18,$ and $18 ÷ 3 = ?$</td>
<td>$? \times 6 = 18,$ and $18 ÷ 6 = ?$</td>
</tr>
<tr>
<td>Equal Groups</td>
<td>If 18 plums are shared equally into 3 bags, then how many plums will be in each bag?</td>
<td>If 18 plums are to be packed 6 to a bag, then how many bags are needed?</td>
</tr>
<tr>
<td>$ Measurement example. You need 3 lengths of string, each 6 inches long. How much string will you need altogether? $</td>
<td>$ Measurement example. You have 18 inches of string, which you will cut into 3 equal pieces. How long will each piece of string be? $</td>
<td>$ Measurement example. You have 18 inches of string, which you will cut into pieces that are 6 inches long. How many pieces of string will you have? $</td>
</tr>
<tr>
<td>Arrays, $^2$ Area</td>
<td>If 18 apples are arranged into 3 equal rows, how many apples will be in each row?</td>
<td>If 18 apples are arranged into equal rows of 6 apples, how many rows will there be?</td>
</tr>
<tr>
<td>$ Area example. What is the area of a 3 cm by 6 cm rectangle? $</td>
<td>$ Area example. A rectangle has area 18 square centimeters. If one side is 3 cm long, how long is a side next to it? $</td>
<td>$ Area example. A rectangle has area 18 square centimeters. If one side is 6 cm long, how long is a side next to it? $</td>
</tr>
<tr>
<td>Compare</td>
<td>A blue hat costs $6. A red hat costs 3 times as much as the blue hat. How much does the red hat cost?</td>
<td>A red hat costs $18 and that is 3 times as much as a blue hat costs. How much does a blue hat cost?</td>
</tr>
<tr>
<td>$ Measurement example. A rubber band is 6 cm long. How long will the rubber band be when it is stretched to be 3 times as long? $</td>
<td>$ Measurement example. A rubber band is stretched to be 18 cm long and that is 3 times as long as it was at first. How long was the rubber band at first? $</td>
<td>$ Measurement example. A rubber band was 6 cm long at first. Now it is stretched to be 18 cm long. How many times as long is the rubber band now as it was at first? $</td>
</tr>
</tbody>
</table>

General

| $a \times b = ?$ | $a \times ? = p,$ and $p ÷ a = ?$ | $? \times b = p,$ and $p ÷ b = ?$ |

$^2$The language in the array examples shows the easiest form of array problems. A harder form is to use the terms rows and columns: The apples in the grocery window are in 3 rows and 6 columns. How many apples are in there? Both forms are valuable.

$^3$Area involves arrays of squares that have been pushed together so that there are no gaps or overlaps, so array problems include these especially important measurement situations.
CGI Framework - Problems

- Look at the problem type charts from the DPI unpacking.
- What types are intended for your grade level?
- What are ways you can differentiate the problem types you need to teach?
Questioning to Guide Instruction

When? How? Why?
When do we ask questions?

- to launch a task or begin a math lesson
- during stations/centers
- during partner work
- during independent work
- during whole class or small group lessons
How do we ask questions?

- **Open ended:**
  - Tell me more.
  - Can you explain...
  - What do you notice?

- **Focused on student ideas:**
  - Allow student to maintain ownership by attributing ideas or strategies to them.
  - With a mathematical goal in mind
  - By encouraging them to make sense of their own and others’ ideas

- **Wait time**
Why do we ask questions?

- to encourage students to engage with important mathematical ideas
- to encourage discourse
- to highlight strategies, connections, aha’s
- to address misconceptions
- to build students’ confidence and sense making abilities
- to better understand students’ thinking
- to challenge students’ thinking
- to begin, continue, end a lesson
Teacher Questioning

Watch this video of a 1st grade class working on True/False equations.

Listen for the questions the teacher asks.

- What questions does she ask?
- What is the purpose of her questions?
- What questions does she NOT ask? Why?
- What questions can she ask to dig deeper?
Questioning to dig deeper

Watch this video to identify

- student misconception(s)
- question(s) the teacher might ask next to
  - find out more about the student’s understanding
  - help her make sense of the task
  - push the student to examine her misconception
Talk Moves

● Wait time
● Turn and Talk
● Revoicing: So are you saying...?
● Say more about that.
● Who can repeat?
● Press for reasoning: Why do you think that?
● What do you think about that? Agree/disagree
● Who can add on?
Number Talks to Increase Mathematical Discourse
What are the characteristics of a number talk?

<table>
<thead>
<tr>
<th>Teacher role</th>
<th>Student role</th>
</tr>
</thead>
<tbody>
<tr>
<td>● teacher asks questions</td>
<td>● patient listening to each other</td>
</tr>
<tr>
<td>● teacher records what the children say</td>
<td>● uses precise language</td>
</tr>
<tr>
<td>● asks for clarification/questioning</td>
<td></td>
</tr>
<tr>
<td>● names previous strategies</td>
<td></td>
</tr>
<tr>
<td>● revisited strategies</td>
<td></td>
</tr>
</tbody>
</table>

Addition Video for 3rd grade: $37 + 38$
What is a number talk?

- short, ongoing *daily* routine that provides students with meaningful ongoing practice with computation
- a powerful tool for helping students develop computational fluency because the expectation is that they will use number relationships and the structures of numbers to add, subtract, multiply and divide
- short sessions alongside (but not necessarily directly related to) the ongoing math curriculum
Goals of Number Talks

The primary goal of Number Talks is to develop computational fluency. While developing this computational fluency, students are better able to . . .

- think and reason like mathematicians.
- make connections and look for relationships and thus are engaged in "doing mathematics."
- share their strategies
- clarify and express their thinking
- developing mathematical language
What are the advantages of implementing number talks within your classroom?

- better communication & more precise language
- builds confidence
- variety of strategies
- validates students’ thinking
- builds number sense
- corrects misconceptions
- focus to lesson
- no writing
How do we create this in our classroom?

Classroom Community
- Organization within the math block
- Routines (thumb up for an answer, adding fingers for additional strategies)
- Respect of ideas / strategies within math talk
  - Smaller groups to encourage listening to each other’s strategies.
- Choosing Problems

Number Talk
- Number String
- One Equation - Focus on Strategies
Without numbers (K-1)

- Ten Frames
- Dot cards
- Rekenreks
Number String

2 \times 25
4 \times 25
6 \times 25

9 + 1
9 + 7 + 1
1 + 6 + 9
Developing a variety of strategies

$16 \times 35 \quad 9 + 8$
Fraction Number Strings

Think about a clock to solve the following problems.

1/2 + 1/3  
1/6 + 1/2  
1/6 + 3/12 + 1/4  
1/6 + 1/4 + 7/12  
3/4 + 2/12

Contexts for Learning, Catherine Fosnot
How can Number Talks become an active part of your classroom?

Quick Draws or Quick Builds
How can Number Talks become an active part of your classroom?

Find a reason why each one does not belong.

http://wodb.ca/shapes.html
Question Stems for Number Talks

- How did you think about that?
- How did you figure it out?
- What did you do next?
- Why did you do that? Tell me more.
- Who would like to share their thinking?
- Did someone else solve it in a different way?
- Who else started the problem this way?
- Who else used this strategy to solve the problem?
- What strategies do you see being used?
- Which strategies seem to be efficient, quick or simple?
Implementation

Number Talks (digital resources)

Math Perspectives Information about Number Talks
Incorporating student centered mathematics within your classroom
Standards Activity: Application

Using your standard for your grade level, map out some instructional activities that you might use to develop those understandings.

Things to Consider:
- Questions you might ask
- Number Talks
- CGI Word Problems
- Tasks to use
- Misconceptions to anticipate and how to incorporate activities that address the misconceptions

Example (Facilitator will make a copy, rename with grade level, and share with each person in your group.)
Closure
Resources

- http://wodb.ca/shapes.html
- Illustrative Mathematics
- NRICH